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Research Article

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Sensitivity Analysis of Internally Reinforced Thin-Walled Hollow-Box Beams Subjected to Uncoupled Bending and Torsion

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Abstract: In this work, novel types of internally reinforced hollow-box beams were subjected to bending loading and studied using the finite element analysis software ANSYS. A parameterization of 3 geometric variables was performed, and deflection and effective deflection results were collected from 2 points at the model. The sensitivity analysis results are then discussed, with the aim of concluding if the selected design variables are adequate for optimization purposes.

Keywords: finite element method, ANSYS, structural static analysis, sensitivity analysis

1 Introduction

Projects related to industrial equipment have suffered severe changes, mainly because of the increasing accelerations caused by electrical motors. Those motors are able to induce movements where acceleration levels can be a magnitude higher the acceleration of gravity. This change in the accelerations makes necessary an improvement in the resistance and stiffness of structures. In fact, the stiffness property of a machine part is one of the basic factors that determine the working capability of equipment and usually is more important than the resistance one in relation to the structure dimensions. The increase in deflection because of high accelerations can cause problems on the equipment's regular behavior. The lack of stiffness in structures causes an increase in friction and wear in the mobile parts, but excessive vibrations remain the main problem, sometimes disturbing smooth operation. A research regarding sensitivity analysis for the same application of an industrial machine was performed earlier by one of the authors, [1]. Optimization for structural applications was also performed in [2]. The authors optimized tubular beams for industrial machine applications in both these works. Silva and Meireles presented the results of an example of 24 optimized beams under uncoupled bending and torsion loadings [3]. The same authors have performed a feasibility analysis of similar beams, which shows that the studied beams are highly effective under bending loadings [4, 5]. Several articles were found in the literature regarding similar beams, manufactured and experimentally tested. Niu *et al.* studied the elastic buckling behavior of rectangular webs for thin-walled beams under a transverse load [6]. Other authors also studied similar topics in [7, 8]. A finite element having seven degrees of freedom at each node was developed for accurate prediction of the stability problem of thin-walled fiber reinforced plastic (FRP) structural members [7]. Liu and Glass studied two automotive thin-walled parts and the effects of wall thickness and geometric characteristics in terms of their structural response [9]. Smyczynski and Magnucka-Blandzi studied the static and dynamic stability of a five-layered sandwich beam subjected to axial compressive force [10]. Shin *et al.* performed a finite element beam analysis of tapered thin-walled box beams subjected to out-of-plane loads and twisting moments [11]. Wang *et al.* discussed bending resistance of thin-walled multi-cell square tubes [12].

The sensitivity analysis of the model studied in this work is very important for optimization purposes. In fact, it determines if the chosen geometric variables are feasible for improving the mechanical behavior of the studied parts when subjected to optimization routines.

2 Improving Structural Behaviour

Nowadays, in many mobile parts applications, such as laser cutting machines and plotters, the accelerations can

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reach values more than 10 times the acceleration of gravity. Therefore, there is a need of improving stiffness of a beam part to allow acceptable correctness of a machine operation without creating any danger of undesired vibrations that can ultimately lead to failure; this requires attention from designers. Equations of the inertia moment of solid parts that can be used for description of behavior of such elements have been proposed by Orlov [13]. Accuracy in calculations of the inertia moment is important for the determination of the strength behavior, especially when considering the computational optimization processes applied to the discussed element. In cases, if the inertia moment calculations lack accuracy, optimization capabilities may be drastically decreased. Although for simple geometries, such a calculation is rather straightforward, but for complex shapes, especially for those composed of several structural elements, the contribution of the inertia moment of each structural element on the global behavior of the part may not be easily determined.

Different ways of improving the mechanical behavior of engineering parts are used in practice. One of them goes through selecting a material having higher Young's modulus value. The other is to modify the inertia moment of a considered part [1, 2]. This can be achieved by inserting ribs in the longitudinal and/or transversal directions and/or webbing. If reinforcements are oriented in the normal direction as of the longitudinal (z-axis) direction, it appears to be effective under bending loads, as they are directed in the same route as bending stresses develop. If reinforcements are oriented along the transversal (xy-plane) direction, they are useful under torsion loading, as the shear stresses arise in the same direction. Orlov studied the effect of ribbing (Figure 1) in terms of improvement in the inertia moment and resistance moment [13]. According to his results, the improvement in the inertia moment measured as I/I_0 and moment of resistance expressed as W/W_0 depend solely on two geometric parameters defined as

$$\delta = \frac{b}{b_0} \quad \text{and} \quad \eta = \frac{h}{h_0},$$

which give

$$\frac{I}{I_0} = 1 + \delta\eta^3 + 3\delta\eta(1+\delta\eta)\left[\frac{1+\eta}{1+\delta\eta}\right]^2 . \tag{1}$$

Figure 1 shows the geometric parameters of the applied rib design: b, b_0, h, and h_0.

The technique of ribbing to improve the structure's inertia moment is commonly used in manufacturing of structures obtained by casting. However, its use has been extended also to plastically formed structures, which can be screwed, welded, riveted, or joined with other forms.

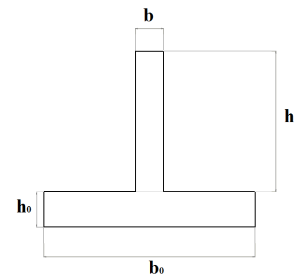

Figure 1: Geometric parameters used to calculate δ and σ

The improvement in the moment of resistance W/W_0 has a correlation with the improvement in the inertia moment, and it is also a dependent of η and δ and is expressed as [13]

$$\frac{W}{W_0} = \frac{I}{I_0}\frac{1+\delta\eta}{1+2\eta+\delta\eta^2} . \tag{2}$$

Figure 2 shows the calculated improvement in the inertia moment using expression (1) in the function of δ and η. The inertia moment value of the cross-section is very important for the stiffness behavior under transverse loads because the deflections directly depend on it. Under bend-

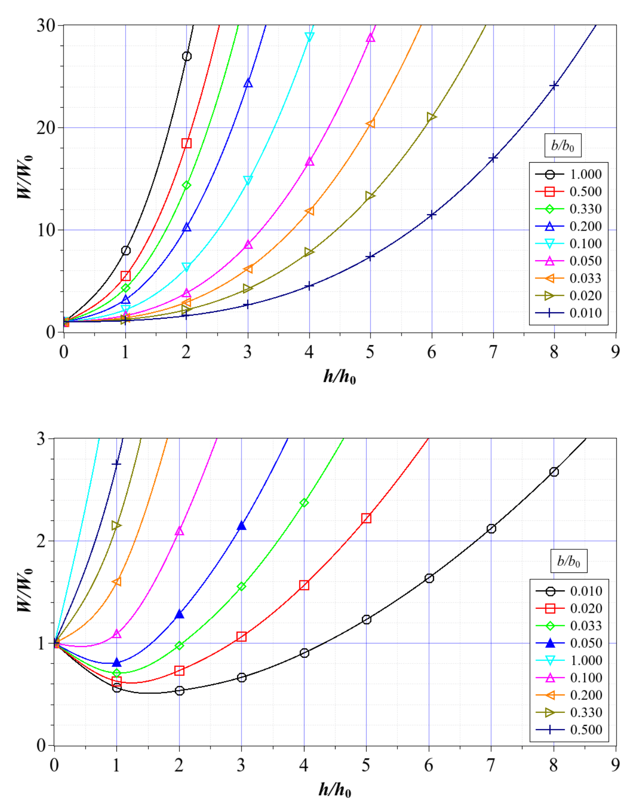

Figure 2: Graphs of I/I_0 (top) and W/W_0 (bottom), calculated using the same expression as a function of δ and η

ing loads, there is an inverse relation between the inertia moment and the deflections.

3 Numerical Procedure

3.1 FEM model

The finite element method (FEM) model of those whose feasibility was already studied in [4, 5] was used here for the sensitivity analysis. The model was based on sandwich geometries whose effectiveness was assessed earlier in [14–16] and is shown in Figure 3. The model is subjected to bending loadings, as this is the relevant form for the target application. Figure 3a shows the FEM model designed, and Figure 3b shows it without the top area to allow the inner view mainly of the transversal reinforcements that are most characteristic.

In order to obtain an effective response to transversal beam load in terms of strength, 24 different FEM models were built in the commercial FEM program ANSYS. The novel beams are composed of two sandwich panels on the top and on the bottom and a reinforcement pattern on the sides. Additional internal reinforcements on one of the beams of the transversal locations are shown in Figure 3 for a chosen example.

The material properties used in ANSYS Mechanical APDL were chosen as for steel: Young's modulus is equal to $2.1 \cdot 10^{11}$ Pa, Poisson coefficient is 0.29, and the material density is 7,890 kg/m^3. As a simplification, the material is considered to be isotropic. Young's modulus and Poisson coefficient are needed in order to obtain the results, and the density is needed for calculating total mass of the object, which is the sum of the masses of its elements. The applied element was SHELL63 (Shell Elastic 4 nodes). They are free quadrilateral elements with a mean length of 0.0025 m. The mesh is fine enough to obtain results converging to the final value. The beam was constrained in the lines of the extremities (z = 0 and z = 1), with the simple support at its ends and is shown in Figure 4.

A concentrated bending load of 1,500 N was modeled in order to simulate the action of bending. This load was applied on the top face, as shown in Figure 4a. A concentrated torsion binary load of 2,000 N was applied in order to simulate the action of torsion. This load was applied on the top face, as shown in Figure 4b.

The results were values of deflection in the direction of the section height, which is that of y-axis, measured in points P1 and P2 shown in Figure 5. The average absolute value between the results collected from these two

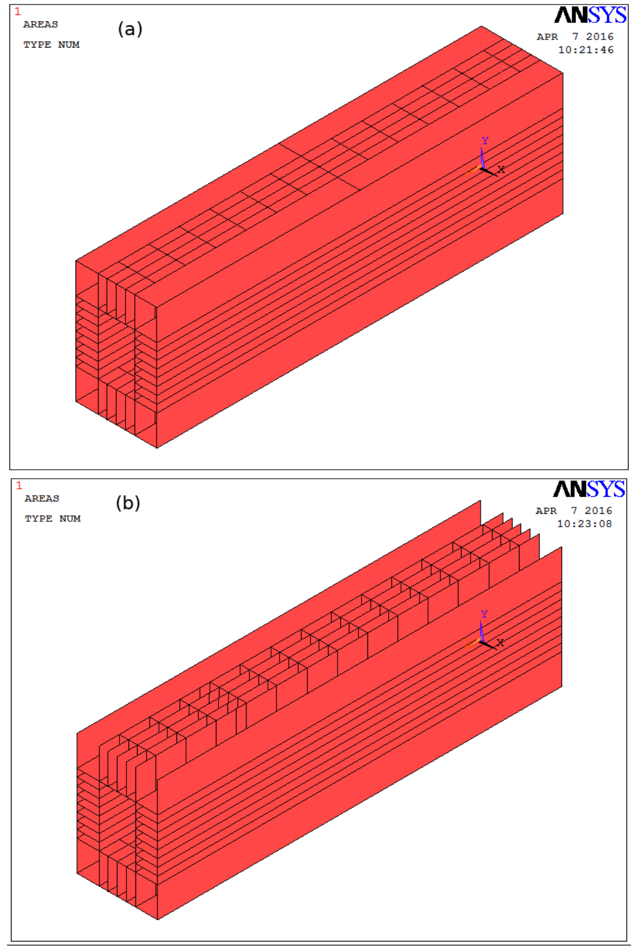

Figure 3: FEM model (a) and FEM model without the top area, to allow the visualization of the transversal reinforcements (b)

points was calculated in each case for any of 24 beam designs. The global maximum is less relevant than local values because of the fact that in the practical application, the loads may be more distributed than that in this case. The maximum absolute values could be taken instead, but the comparison would be erroneous because the novel beams are of much lower thickness and the effect of the concentrated load is amplified than it would be in the respective HSS (Hollow Solid Section) beams, which were chosen as a base for the improvement level calculations.

These points were chosen in places where all coordinates are kept the same, in spite of the variation of the geometric variables. This avoids the direct influence of the change in the values of design variables on the results.

Table 1: Model mass and value of the studied geometric variables

Variables and Mass	j = 1	j = 2	j = 3	j = 4	j = 5
LG1	0.035	0.04	0.045	0.05	0.055
mass (kg)	82.762	81.567	80.372	79.176	77.981
LG2	0.065	0.07	0.075	0.08	0.085
mass (kg)	82.346	81.359	80.372	79.384	78.397
LG3	0.002	0.0025	0.003	0.0035	0.004
mass (kg)	53.581	66.976	80.372	93.767	107.162

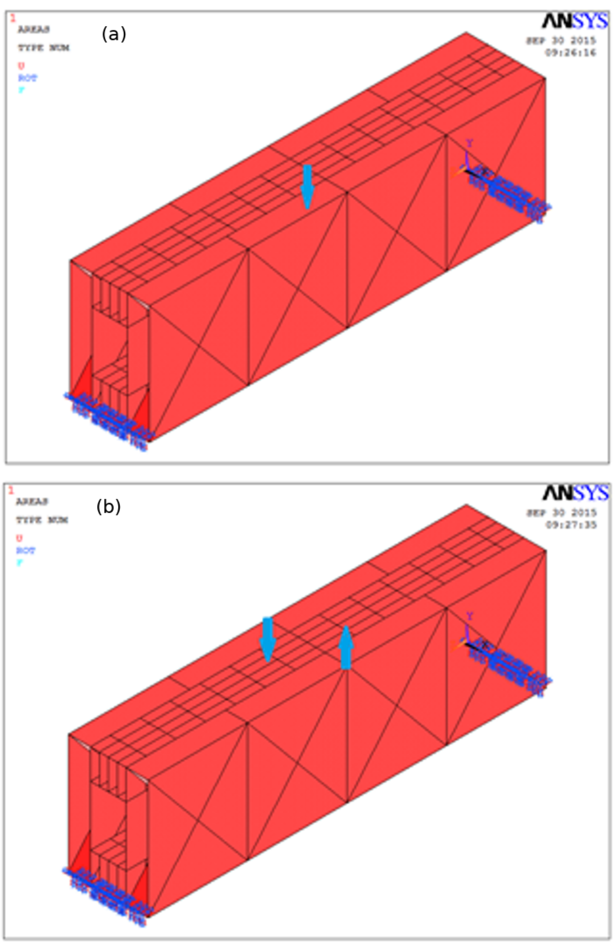

Figure 4: DOF Constraints and loadings in bending [3–5]

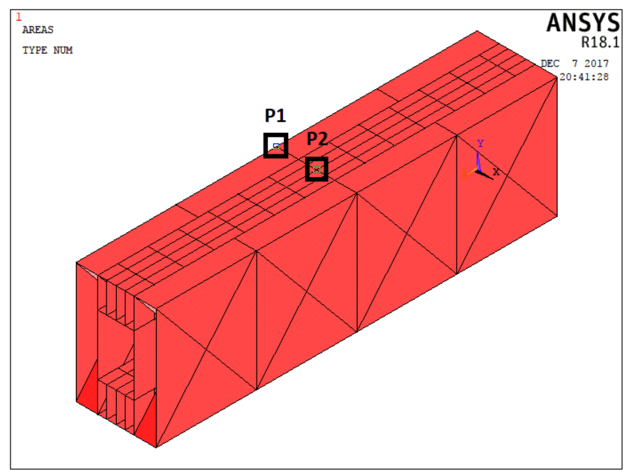

Figure 5: Points used to calculate displacements on sensitivity analysis and on the calculation of the objective function on optimization procedure [3–5]

4 Methodology

To evaluate the sensitivity of the models to the chosen parameters, which were the dimensions of LG1, LG2, and LG3 shown in Figure 6:

LG1: half of the length of the x-dimension of the inner beam;
LG2: half of the length of the y-dimension of the inner beam;
LG3: thickness of the object.

The results were evaluated by means of the FEM model, as shown in Figure 3. It was run on ANSYS MECHANICAL APDL, and one of the abovementioned variables at a time had its value changed. The total mass and y-displacement value in the three points shown in this figure were collected. The results are shown in Figures 7–10.

The results shown in Figures 7–10 were created from data collected from ANSYS MECHANICAL APDL by running the ANSYS input file each time after modifying the value of one variable at each time. The other geometric variables are kept constant. However, when varying one geometric variable, the mass always varies. In order to collect the results, the keypoints that are located on the edge and on the center, are chosen, see Figure 5. The coordinates of the keypoints do not change during the optimization process with the change in the variable values. These points are strongly reinforced with ribs, and, as such, it is not expected that the local deformation because of the low thickness to be significant for other considered thicknesses. The outer section dimensions are kept, by princi-

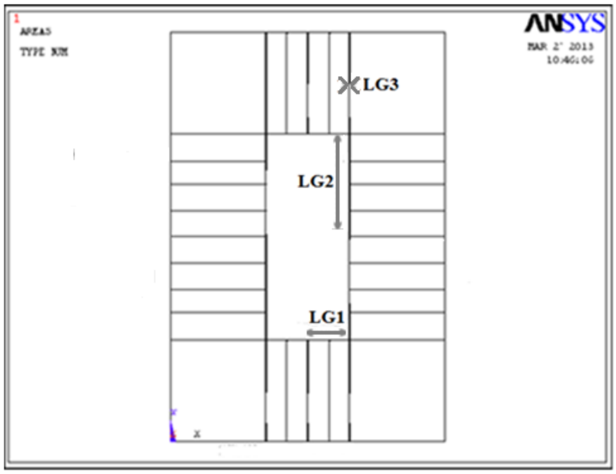

Figure 6: Geometric variables of the FEM model

ple, unaltered. It is assumed that, from an industrial point of view, all beams shall be constructed from a steel sheets of the same thickness. The aim is to obtain a set of reinforcements that are industrially easy to assemble. The values of the variables as well as the resulting model mass are presented in Table 1.

5 Results

5.1 Deflections

The variation in all considered variables (LG1,LG2 and LG3) for bending load is shown in Figure 7 for point P1 and in Figure 8 for point P2 in terms of deflections. These results indicate that the LG3 variable value has most influence to the calculated absolute deflection values, both in points P1 and P2, and torsion is definitely dominant over bending.

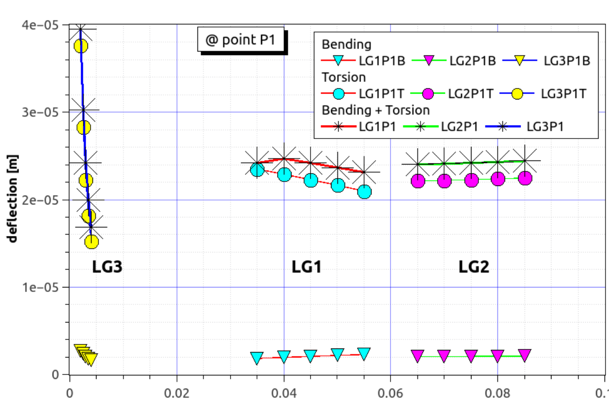

Figure 7: Absolute deflections at point P1

Changes in lengths of LG1 and LG2 result in minor variation in deflection in both cases of loads, bending and torsion. For all LG variables, torsion is dominant and bending adds a little to the combined state of loading. Increase in the length of the LG3 from 0.002 m by 100% to 0.004 m causes the deflection in point P1 to diminish from $4e^{-5}$ m by about 60% to $1.7e^{-5}$ m. It should be noted that 90% of the deflection is caused by torsion. This is due to the fact that all of the internal reinforcements are oriented along the longitudinal axis, which leads to high bending stiffness, but torsional stiffness of the structure is not so high. To improve torsional stiffness reinforcements oriented perperdicularly to the longitudinal axis (z-axis), that is, xy plane should be added before design for practical application is done. Also, point P1 is located at the edge and the area right below is not reinforced. This causes local weakness of the structures, which is very important for the case of point loads, such as in the case of the present work.

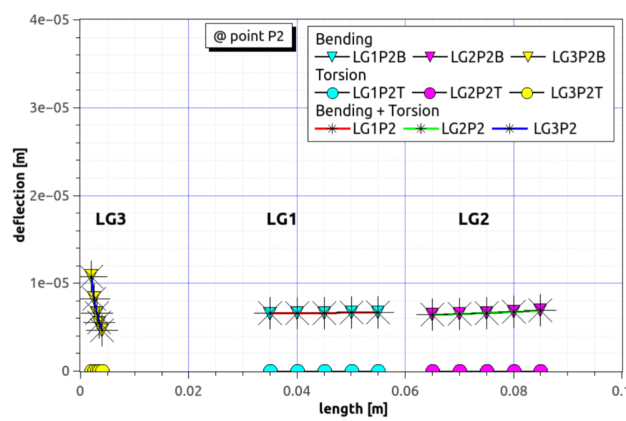

Figure 8: Absolute deflection at point P2

In case of point P2, all absolute deflections are much smaller being less than half of those in point P1, Figure 8. This can be explained by the fact that point P2 is located right above a web-core sandwich panel, which is a very effective structure for transverse loadings, provided that the reinforcements are oriented in the direction of the sensitive plane for the loads that are being applied. In this case, this is true for the case of bending loads. Similarly, LG3 possess highest dynamics of changes in deflection; however, in this case, bending dominates over torsion whose influence is close to zero. LG1 and LG2 results are flat and also their combined values are caused by bending. For all variables, average level of deflections is similar and smallest value of deflections is noted for the largest dimension of LG3 where it is about a half of those for LG1 and LG2.

5.2 Effective Deflection

In Figures 9 and 10, the effective deflection values detected at both the measuring points P1 and P2, respectively, are presented. The effective deflection is considered as the deflection value multiplied by the total mass of the model.

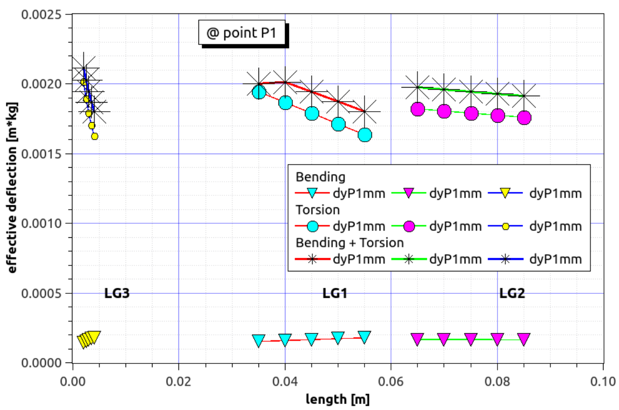

Figure 9: Effective deflections for measuring point P1

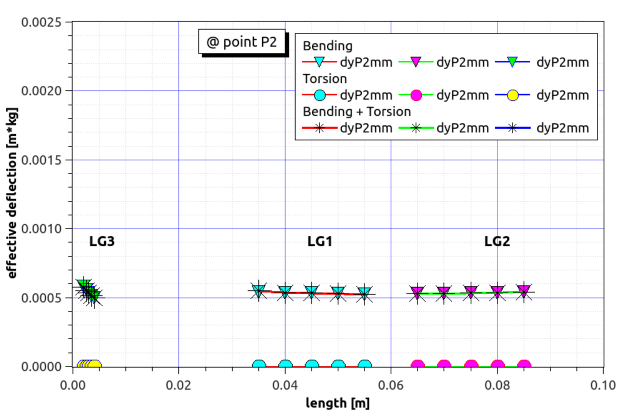

Figure 10: Effective deflection for measuring point P2

When mass of the model is taken into account, the character of the results is slightly different when compared to the absolute deflections. For point P1, the dynamics of changes for the LG3 value diminishes and is similar to that for LG1 and LG2. All of them are decreased with an increase in their respective dimensions where even all their final values are similar. Again, the dominant state of load for results is torsion for every considered LG values. Bending causes not more than 12% of effective deflection, and this is slightly more than that for the absolute deflections. LG2 changes have smallest influence on the results. Again, this is due to point P1 being unreinforced, which causes the height of the reinforcements (LG2 variable) not to be very relevant in structural terms for that measuring point.

For point P2, dynamics of results flattens, including the mass of the model significantly diminishes changes in effective deflections. Comparison to those at point P1 shows not more than 25% in numbers and bending is dominating while torsion remains at zero level. This happens due to the fact that point P2 is the point of load application in case of bending. As such, it is the point that is more solicited mechanically. In the case of torsion, it is the point whose deflection is negligible.

5.3 Results Discussion

Two metrics were used here for collecting all results: deflection and effective deflection. The metric deflection is the *y*-deflection of the model for the studied conditions. Often, the results generated from this measure are logical and, for most cases, are represented by a strict variation.

However, from the design point of view, when both mass and deflections matter, another measure should be used. That measure is the *y*-deflection multiplied by the mass. It is called "effective deflection." It often generates unexpected results, with non-strict decrease or increase. However, it makes possible to determine, in most cases, design considerations, such as number of reinforcements. The best conditions are those that generate the lower effective deflection for the lowest value of the independent variable, for example, number of ribs.

In this work, the choice of the measuring points P1 and P2, as shown in Figure 5, is not based on any other considerations than those points being the ones with highest deflection under bending (P2) and under torsion (P1).

For all variables, the results are shown in the same scale of charts. Despite the fact that the thickness and its variation magnitude is lower (LG3 variable) than the other two variables: LG1 and LG2, for all the results, presented in Figures 7–10, the LG3 variable is the one that the studied model is more sensitive to. The measuring point P1 is the one that generates higher variation on the results, in the case of torsion and also under bending loadings. The variables LG1 and LG2 generate similar variation on the results for both deflection and effective deflection. The variable LG3 (thickness) generates a much sharper variation on the results for both shapes. For bending and torsion coupled loading, it should be noted that LG3 is also quite more sensitive than LG1 or LG2. The effects of obtained at important points P1 and P2 are more balanced, as expected, because the contributions of both bending and torsion are combined in the finite element software ANSYS.

6 Conclusions

Under bending loadings, deflections and effective deflections for all cases are dominant at point P2. Under torsion loadings, deflections and effective deflections for all cases are most important at point P1. This can be explained by the characteristics of the loading mode in the two cases.

In the case of bending, it is possible to conclude that

- The model is sensitive to the variation of the geometric variables. The chosen geometric variables are, therefore, relevant to optimization purposes with LG3 potentially of highest importance.
- The effective deflection does not always vary in the same manner as the deflection itself. This measure, as shown in Figures 7–10, is more representative to the structure effectiveness.
- The three studied variables do not cause similar results what is caused by loads applied at one side only. Therefore, the variables LG1 and LG2 must show different results than LG3, due to their different positions but their influence remains the most important.

In the case of torsion, the following conclusions can be drawn from this work

- The model is sensitive to some of the variation in the geometric variables. Deflections in numbers are caused by such loading type, which is most important to the discussed beam.
- The variable LG3, representing thickness of the element, shows different results in comparison to the two other variables LG1 and LG2, which are related to dimensions, because of its scope being global.

In general terms, it is possible to conclude that

- The stiffness behavior always show a strict variation. However, from the point of view of practical application in which lightweight and stiff parts are desirable, the effective deflection is more relevant.
- Although a linear (elastic) material model was used, the effective deflection does not always show strictly linear relationship with any of the design variables. Nevertheless, even for the non-strict variations, the trend of the results is clear.
- Effective deflection and different trends because of LG variables allow to indicate that implementation of optimization routines may be able to generate a more effective solutions.
- Maximum deflection obtained is close to 0.05 mm, and happens for the variable LG3, under torsion measured at point P1. Such deflection compared to the overall section height where it was calculated, is only 0.0167%. This relates to static deflection. It would be worth finding how this aspect would look at dynamic conditions and at what natural frequency.
- Implementation of the effective deflection in optimization purposes may generate some non-trivial results, both in terms of the values of design variables LG1, LG2, and LG3 as well as the objective function, and even the mechanical behavior of the optimized parts.
- Implementation of the studied models, variables, and effective deflection into an optimization routine may lead to the design of highly effective parts for engineering applications in which lightweight and stiff properties are of great importance.
- In all cases, for both deflection and effective deflection, the studied structure is significantly less stiff at point P1 than at point P2. In design considerations, internal reinforcement should be performed more effectively along the longitudinal axis of the beam (z-axis), across the midspan of the axis of the width (x-axis). This can be performed by adding thicker reinforcements or by increasing reinforcement density close to point P1. This design consideration may be able to significantly improve the effective behavior of the studied model, as well as similar structures.
- As such, optimization of the reinforcements in terms of optimizing reinforcement density and thickness at critical point P1 as well as its surroundings could greatly improve the performance of the studied machine element, as well as similar structures, in practical applications.

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